(or spiked). Factor B represents different dilutions of the spiked sample. The measured response is the $\log_{10}$ of the plaque forming units per mL of solution.

Since factor A (sample) is not a quantitative factor it would be inappropriate to use orthogonal polynomial contrasts to partition its sums of squares or the sums of squares of its interaction with factor B (Dilution). To determine if the additive model $y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}$ is appropriate for this data, test to see whether there is a significant interaction using Tukey's method. The function `Tukey1df` in the R package `daewr` calculates the non-additivity or interaction sums of squares, shown in Equation (3.7), and prints a report. The code to open the data in Table 3.5 and call the function are shown below. The first column in the data frame used by this function is a numeric response, the second column is the indicator for the factor A, and the third column is the indicator for the factor B. The number of rows in the data frame should be exactly equal to the number of levels of factor A times the number of levels of factor B, since the design has no replicates.

```
> library(daewr)
> Tukey1df(virus)
Source           df     SS        MS         F      Pr>F
A                 5   0.1948    0.039
B                 2   3.1664    1.5832
Error            10   0.1283    0.0513
NonAdditivity     1   0.0069    0.0069    0.51    0.4932
Residual          9   0.1214    0.0135
```

In the results, it can be seen that the interaction (or non-additivity) is not significant. Therefore, for this data, it would be appropriate to fit the additive model, $y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}$, with the R function `lm` or `aov`.

3.6 Factorial Designs with Multiple Factors—CRFD

Two-factor factorial designs are more efficient than studying each factor separately in one-factor designs. Likewise, when many factors are under study, it is more efficient to study them together in a multi-factor factorial design than it is to study them separately in groups of two using two-factor factorial designs. When multiple factors are studied simultaneously, the power for detecting main effects is increased over what it would be in separate two-factor factorial designs. Also, the possibility of detecting interactions among any of the factors is possible. If the factors were studied separately in two-factor factorials, two-factor interactions could only be detected between factors studied together in the same design. In a multi-factor factorial not only is it possible to detect two-factor interactions between any pair of factors, but it is also possible to detect higher order interactions between groups of factors. A three-factor interaction between factors A, B, and C, for example, means the effect of factor A differs depending on the combination of levels of factors B